17-01-2019. Thursday.

Electromagnetic waves:- ($B_1 \neq 0$; $B_0 = 0$).

Electric field oscillations will give electro-magnetic oscillations so; $B_1 \neq 0$.

Example: Light waves, microwaves. These are high frequency waves (electron waves).

Maxwell's Equation:

$$\bar{\nabla} \times \bar{B}_1 = \frac{4\pi}{c^2} J_1 + \frac{1}{c^2}\frac{\partial E_1}{\partial t} \quad \text{--- (1)}$$

Electric and magnetic fields are always perturbed so, we cannot write B_0.

$$\bar{\nabla} \times \bar{E}_1 = -\dot{B}_1 \quad \text{--- (2)} \qquad \because \bar{J} = neV.$$

Differentiating eq. (1) w.r.t time,

$$\bar{\nabla} \times \dot{B}_1 = \frac{4\pi}{c^2} \dot{J}_1 + \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2} \quad \text{--- (3)}$$

Substituting eq. (2) into eq. (3)

$$-\left[\bar{\nabla} \times (\bar{\nabla} \times \dot{E}_1)\right] = \frac{4\pi}{c^2} \dot{J}_1 + \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2}$$

$$-\left[\bar{\nabla}(\bar{\nabla} \cdot \bar{E}_1) - \nabla^2 E_1\right] = \frac{4\pi}{c^2} \dot{J}_1 + \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2}$$

$$-\left[ik\,(ik \cdot \bar{E}_1) + k^2 \bar{E}_1\right] = \frac{4\pi}{c^2}\left(-i\omega J_1\right) + \frac{1}{c^2}\left(-\omega^2 E_1\right)$$

Date:

$$(\omega^2 - c^2k^2)\bar{E}_1 = -4\pi i\omega J_1 \quad \text{---- (4)}$$

Electromagnetic waves propagating through vaccum then J_1 will be zero.

So,

$$(\omega^2 - c^2k^2)E_1 = 0$$

$$\omega^2 = c^2k^2 \quad ; \quad E_1 \neq 0.$$

This is the dispersion relation of light waves through vaccum or photons.

- Photon in vaccum act as a wave.

Dispersion relation in medium:-

If we have a medium then

J_1 = current density $= neV = n_0 e(-V_{e1})$

$$J_1 = -n_0 e V_{e1} \quad \text{---- (5)}$$

Equation of motion for electrons,

$$m\cancel{n_0}\frac{\partial V_{e1}}{\partial t} = -\cancel{n_0}e\bar{E}_1$$

$$\frac{\partial V_{e1}}{\partial t} = -\frac{e\bar{E}_1}{m}$$

$$\cancel{-}i\omega V_{e1} = \cancel{-}\frac{eE_1}{m}$$

$$V_{e1} = \frac{e\bar{E}_1}{i\omega m}$$

$$V_{e1} = -\frac{ie\bar{E}_1}{m\omega}$$

Substituting it in eq. (5),

$$J_1 = -n_0 e V_{e1}$$

$$= -n_0 e\left(-\frac{ie\bar{E}_1}{m\omega}\right)$$

$$J_1 = \frac{in_0e^2\bar{E}_1}{m\omega}$$

Put it in eq. (4), we get

$$(\omega^2 - c^2k^2)\bar{E}_1 = -4\pi i\omega J_1$$

$$(\omega^2 - c^2k^2)\bar{E}_1 = -4\pi i \omega \left(\frac{i n_0 e^2 \bar{E}_1}{m\omega}\right)$$

$$= -\frac{4\pi i^2 n_0 e^2}{m}$$

$$\omega^2 - c^2k^2 = \frac{4\pi n_0 e^2}{m}$$

$$\omega^2 = \frac{4\pi n_0 e^2}{m} + c^2k^2$$

$$\omega^2 = \omega_{pe}^2 + c^2k^2 \qquad \because \omega_{pe}^2 = \frac{4\pi n_0 e^2}{m}$$

This is the dispersion relation for Electromagnetic waves in plasma medium.
Photons in medium act as a particle as well a wave.
It shows dual nature in plasma medium.
Different harmonics are there in Electromagnetic waves.
It propagate in the form of envelop.
Electromagnetic wave is the only wave that has velocity greater than velocity of light.

Phase Velocity:-

$$V_\phi = \frac{\omega^2}{k^2} = c^2 + \frac{\omega_{pe}^2}{k^2}$$

V_ϕ of Electromagnetic wave is more than velocity of light.

Group Velocity:-

$$\omega^2 = \omega_{pe}^2 + c^2k^2$$

Differentiating above equation,

$$\cancel{2}\omega d\omega = \cancel{2}c^2k\,dk$$

$$\frac{d\omega}{dk} = \frac{c^2k}{\omega}$$

Scanned with CamScanner

$$\frac{d\omega}{dk} = \frac{c^2}{\omega/k}$$

$$\frac{d\omega}{dk} = \frac{c^2}{V_\phi}$$

Group velocity is less than the velocity of light.

23-01-2019. Wednesday.

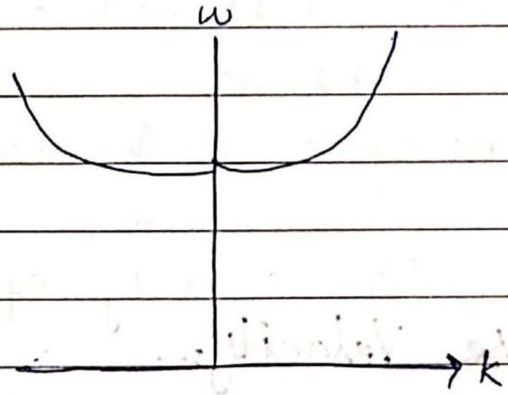
$$\omega^2 = \omega_{pe}^2 + c^2k^2 \quad \text{— i)}$$

Plasma Electron wave

$$\omega^2 = \omega_{pe}^2 + \frac{3}{2}k^2 V_{th}^2 \quad \text{— ii)}$$

If we compare eq. i) & ii), these two are almost similar but the only difference is that in Electro-magnetic wave we have speed of light "c" & in electron wave we have thermal velocity "V_{th}".

$V_{th} << c$



$$\omega^2 = c^2k^2 + \omega_{pe}^2$$

$$k^2 = \frac{\omega^2 - \omega_{pe}^2}{c^2}$$

i) If we increase ω_{pe}^2 so that its value becomes equal to ω^2 then k is zero & wave will not propagate. Wave will only oscillate.

$$\omega_{pe}^2 = \frac{4\pi n_0 e^2}{m_e}$$

ii) When $\omega > \omega_{pe}$ the "k" is positive & wave will ~~prog~~ propagate.

iii) If $\omega < \omega_{pe}$ then k is imaginary.

Date:

Skin Depth:-

i) $\omega > \omega_{pe}$ then $k = +ve$

ii) $\omega = \omega_{pe}$ then $k = 0$

iii) $\omega < \omega_{pe}$ then $k =$ Imaginary.

We can write

$$k^2 = -\frac{(\omega_{pe}^2 - \omega^2)}{c^2}$$

$$k = \sqrt{-\frac{(\omega_{pe}^2 - \omega^2)}{c^2}}$$

$$k = \sqrt{\frac{i^2(\omega_{pe}^2 - \omega^2)}{c^2}}$$

$$k = \frac{i\sqrt{\omega_{pe}^2 - \omega^2}}{c}$$

- Skin depth means how plasma wave penetrate deep inside the skin. When "k" is imaginary then it is called skin depth.

$$\text{Skin depth} = \frac{2\pi}{\lambda} = \frac{i\sqrt{\omega_{pe}^2 - \omega^2}}{c}$$

$$\text{Skin depth} = \lambda_s = \frac{c}{2\pi\sqrt{\omega_{pe}^2 - \omega^2}}$$

$\therefore$ Imaginary k = Skin depth $= \frac{2\pi}{\lambda_s}$